

PRIFYSGOL CYMRU : UNIVERSITY OF WALES

**DEGREE EXAMINATIONS 2012
ARHOLIADAU GRADD 2012**

**Swansea Metropolitan University
Prifysgol Fetropolitan Abertawe**

Faculty of Applied Design and Engineering

NAME OF COURSE/PROGRAMME: *BEng Motorcycle Engineering*

YEAR or LEVEL (state which): *Year III*

NAME OF MODULE/UNIT: *Advanced Motorcycle Dynamics*

NAME OF EXAM PAPER:
(if different to module name)

DATE OF EXAMINATION:

EXAMINATION TIMES:

Start Time:
Finish Time:
Time Allowed:

INSTRUCTIONS TO CANDIDATES:

For all questions, take $\rho_{air} = 1.167 \text{ kg m}^{-3}$

An equation sheet is provided with this exam paper

Answer 3 from 4 Questions

FACULTY REF:

PAGE 1 OF 6

Question 1

Consider a simple mass m attached to a spring of stiffness k and a viscous damper with a damping coefficient c . The displacement, x , is given by:

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = 0$$

- a. Show that this equation is mathematically the same as:

$$\frac{d^2 x}{dt^2} + 2\zeta\omega \frac{dx}{dt} + \omega^2 x = 0$$

(4 marks)

- b. Solve this equation for $\zeta = 1$

(6 marks)

- c. The equation of motion for a mass-spring-damper system is given by the equation:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = 0$$

For $\zeta < 1$ and the initial conditions, when $t = 0$, $x = 0$ and $\dot{x} = \beta\omega$

Where $\beta = \sqrt{1 - \zeta^2}$, show that $x = e^{-\zeta\omega t} \sin(\beta\omega t)$

(10 marks)

Question 2

The motorcycle in its plane of symmetry can be represented as three rigid bodies whose vibrating motion is described by four independent coordinates:

- The vertical displacement of the sprung mass centre.
- The pitching rotation of the sprung mass.
- The vertical displacements of the *two* unsprung masses.

a. Draw a schematic representation of the system.

(6 marks)

b. Show that the equations of free, undamped motion can be represented in the form

$$ma + kx = 0$$

described in a 4x4 matrix

(14 marks)

Question 3

The motorcycle has two main types of vibrational modes:

The in-plane and out-of-plane modes of vibration for a motorcycle are affected by different parameters.

- a. Explain what influences the in-plane vibrational modes and what effect this has on the motorcycle and the rider.

(3 marks)

- b. Explain what parameters influence the out-of-plane modes, and what mathematically determines the natural frequency of the oscillating modes.

(5 marks)

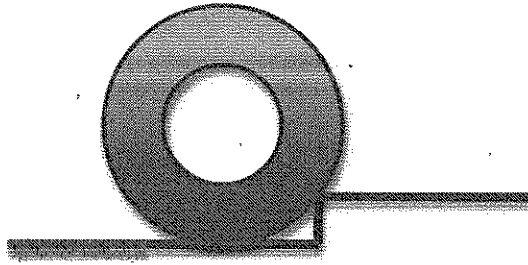
Experimentally, the motorcycle can exhibit a number of different oscillatory and non-oscillatory modes. Define the following terms and describe how they manifest themselves in motorcycle dynamics and how they are modeled mathematically.

- c. Capsize
- d. Wobble
- e. Weave

(12 marks)

Question 4

Consider a wheel climbing a step.



- a. If the radius of the wheel is R and the height of the step is h , find an expression in 's' such that

$$s = \sqrt{h(2R - h)}$$

(4 marks)

- b. As the wheel climbs the step a vertical distance 'y' and by equating the rate of climb to the rate of horizontal travel, show that:

$$\frac{dy}{dt} = V \sqrt{\left\{ \frac{R}{R - (h - y)} \right\}^2 - 1}$$

(10 marks)

The tyre model proposed by Pacejka in 1991 is still very much in use today. The approach is substantially empirical and the results reproduce the real behaviour of the tyre very well. The entire model revolves around what is called a 'magic formula', which is a single expression that can be used to represent the longitudinal driving or braking force, the lateral force or the moment around the z-axis. The expression is:

$$F(x) = \mu(x) + S_v$$

$$\mu(x) = D \sin\{C \tan^{-1}[Bx - E(Bx - \tan^{-1} Bx)]\}$$

$$X = x + S_h$$

Where B , C , D and E are four parameters, S_v indicates the translation of the curve along the y-axis and S_h indicates the translation of the curve along the x-axis.

- c. Sketch the curve of the Pacejka "Magic Formula", accurately labelling the parameters outlined in the expression above.

(6 marks)



EXAMINATION SOLUTIONS : SESSION 2011/12

Programme:	BEng Motorcycle Engineering	Year:	Level 6
Module/Unit:	Advanced Motorcycle Dynamics		
Author:	Dr Owen Williams	Page:	1 of 6

Question No	Question	Marks
1 a	Divide by m $\frac{d^2x}{dt^2} + \frac{c}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$ $= \frac{d^2x}{dt^2} + \frac{c k^{1/2} m^{-1/2}}{k^{1/2} m^{1/2}} \frac{dx}{dt} + \frac{k}{m} x = 0$ $= \frac{d^2x}{dt^2} + \frac{c \sqrt{k}}{\sqrt{km}} \frac{dx}{dt} + \frac{k}{m} x = 0$ $= \frac{d^2x}{dt^2} + \frac{2\zeta \sqrt{km} \omega}{\sqrt{km}} \frac{dx}{dt} + \frac{k}{m} x = 0$ $= \frac{d^2x}{dt^2} + 2\zeta \omega \frac{dx}{dt} + \omega^2 x = 0$ $\omega = \sqrt{\frac{k}{m}}$ $\zeta = \frac{c}{2\sqrt{km}}$	(4)
6	$r^2 + 2\zeta \omega r + \omega^2 = 0 \Rightarrow$ quadratic eq $r = \omega(-\zeta \pm \sqrt{\zeta^2 - 1})$ when $\zeta = 1$ $r = -\omega$ so equal roots for equal roots $x = (A + Bt)e^{-\omega t}$	(6)

Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	2 of 6
--------------	------------------------------	-------	--------

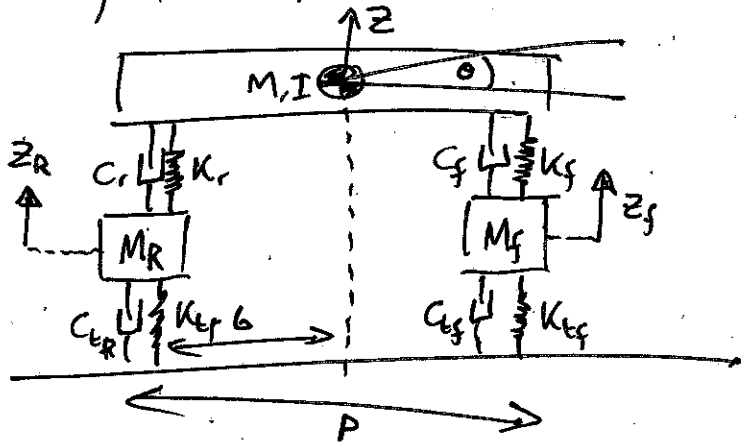
Question No	Question	Marks
1c	This equation is same as previous so notice $r = \omega(-\zeta \pm \sqrt{\zeta^2 - 1})$ But now $\zeta < 1$ so $\zeta^2 - 1$ is negative $\Rightarrow$ Complex roots $r = \omega(-\zeta \pm \sqrt{-(1 - \zeta^2)})$ $r = \omega(-\zeta \pm i\sqrt{1 - \zeta^2})$ Now since $\beta = \sqrt{1 - \zeta^2}$ $r = -\omega\zeta \pm i\omega\beta$ Now as $r = \alpha + i\beta \Rightarrow x = e^{\alpha x} [A \cos(\beta x) + B \sin(\beta x)]$ So $x = e^{-\zeta\omega t} [A \cos(\beta\omega t) + B \sin(\beta\omega t)]$ when $t=0, x=0$ so $0 = e^0 [A \cos 0 + B \sin 0]$ $A = 0$ So $x = B e^{-\zeta\omega t} \sin(\beta\omega t)$ Now $\dot{x}$ $\dot{x} = -\zeta\omega B e^{-\zeta\omega t} \sin(\beta\omega t) + B\beta\omega e^{-\zeta\omega t} \cos(\beta\omega t)$ Now when $t=0, \dot{x} = \beta\omega$ $\beta\omega = 0 + B\beta\omega$ So $B = 1$ Now $A=0, B=1$ $x = e^{-\zeta\omega t} [A \cos(\beta\omega t) + B \sin(\beta\omega t)]$ $x = e^{-\zeta\omega t} \sin(\beta\omega t)$	10 20

Continuation Sheet

Module/Unit: Advanced Motorcycle Dynamics

Page:

3 of 6

Question No	Question	Marks
2a	This 4 DoF system is represented as :  b Resolving forces for undamped: $M\ddot{z} + (K_f + K_r)z + (P-b)K_f\theta - bK_r\theta - K_f z_f - K_r z_r = 0$ $= M\ddot{z} + (K_f + K_r)z + ((P-b)K_f - bK_r)\theta - K_f z_f - K_r z_r = 0$ Rotationally $I\ddot{\theta} + ((P-b)K_f - bK_r)z + ((P-b)^2 K_f + b^2 K_r)\theta - (P-b)K_f z_f + bK_r z_r = 0$ For M_f $M_f \ddot{z}_f - K_f z - (P-b)K_f \theta + (K_f + K_{ff})z_f = 0$ for M_r $M_r \ddot{z}_r - K_r z + bK_r \theta + (K_r + K_{rf})z_r = 0$ This gives $\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & M_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_f \\ \ddot{z}_r \end{bmatrix} = \begin{bmatrix} K_f + K_r & (P-b)K_f - bK_r & -K_f & -K_r \\ (P-b)K_f - bK_r & (P-b)^2 K_f + b^2 K_r & -(P-b)K_f & bK_r \\ -K_f & -(P-b)K_f & K_f + K_{ff} & 0 \\ -K_r & bK_r & 0 & K_r + K_{rf} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ z_f \\ z_r \end{bmatrix} = 0$	(6) (14) 20

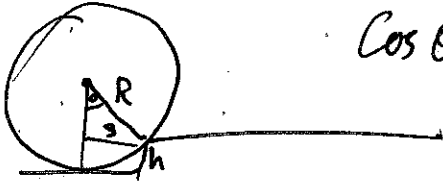
Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	4 of 6
--------------	------------------------------	-------	--------

Question No	Question	Marks
3	The in-plane vibrational modes of the motorcycle are influenced by the frame, suspension and the wheel motions in the vertical plane. The in-plane modes are related to ride comfort and road-holding. The out-of-plane modes of vibration involve the roll, yaw, steering angle and steering head lateral displacement. The out-of-planes modes are generally related to the vehicle stability and handling. The natural frequencies of the oscillating modes (both in and out of plane) are given by the imaginary parts of the eigenvalues. The capsize motion is in essence a roll motion deeply influenced by the rider action on the handlebar. This motion is always unstable as the motorcycle can be thought of as an inverted pendulum. It is modelled mathematically as: $\varphi = \varphi_0 e^{\frac{t}{\tau}}$ Where tau is a positive time constant, showing capsize is always unstable. Wobble is an oscillation of the front assembly around the steering axis that can become unstable at fairly low to mid speeds. Wobble frequency goes up as the trail of the motorcycle increases and as the front-frame inertia decreases. Mathematically this is: $I_{Af} \ddot{\delta} = -c\dot{\delta} - F_{Sf} a_n + M_f g b_f \delta \sin \epsilon + N_f a_n \delta \sin \epsilon$ Weave is an oscillation of the whole motorcycle, but mainly an effect seen at the rear end of the motorcycle. It depends on the CofG of the motorcycle, the wheel inertia, the caster angle and trail and the cornering stiffness of the rear tyre. The weave mode is technically generated by the coalescence of two non-oscillating modes; body capsize and steering capsize. Mathematically it is the solution of: $I_{Ar} \ddot{\psi} = -c\dot{\psi} - F_{Sr} l$	3 12 20

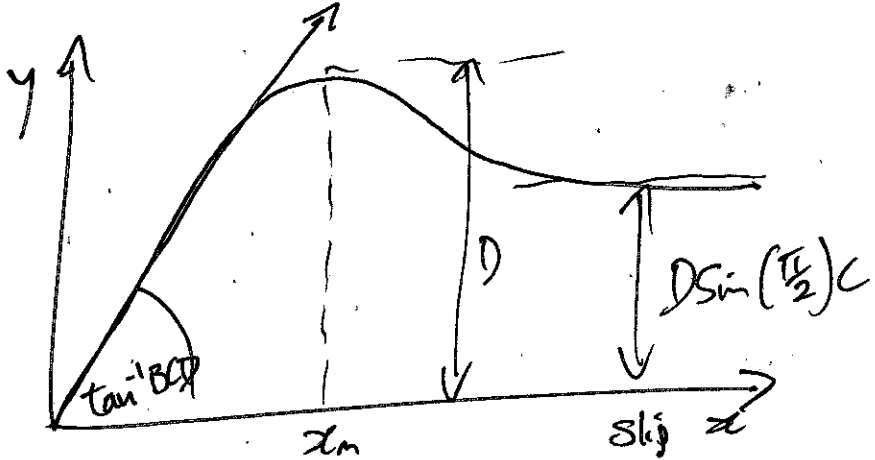
Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	5 of 6
--------------	------------------------------	-------	--------

Question No	Question	Marks
4a	 $\cos \theta = \frac{R-h}{R} \quad \sin \theta = \frac{S}{R}$ $S = R \sin \theta = R \sqrt{1 - \cos^2 \theta}$ $S = R \sqrt{1 - \left(\frac{R-h}{R}\right)^2}$ $S = \frac{R}{R} \sqrt{R^2 - (R-h)^2} = \sqrt{R^2 - R^2 + 2Rh - h^2}$ $S = \sqrt{2Rh - h^2} = \sqrt{h(2R-h)}$	(4)
4b	at a vertical distance up $(h-y)$ S becomes $S = \sqrt{(h-y)(2R-h+y)}$ $S^2 = (h-y)(2R-h+y) \quad \text{differentiate}$ $2S \frac{dS}{dt} = -2R \frac{dy}{dt} + h \frac{dy}{dt} - 2y \frac{dy}{dt} + h \frac{dy}{dt}$ $2SV = 2(h-y-R) \frac{dy}{dt}$ $\frac{dy}{dt} = \frac{SV}{h-y-R} \quad \text{Substitute } S$ $\frac{dy}{dt} = \frac{V \sqrt{(h-y)(2R-h+y)}}{h-y-R} = \frac{V \sqrt{(h-y)(2R-h+y)}}{R-(h+y)}$ $\frac{dy}{dx} = V \sqrt{\frac{2Rh - h^2 + hy - 2Ry + hy - y^2}{(R-(h+y))^2}} = V \sqrt{\frac{R^2 - (R^2 - 2R(h-y) + (h-y)^2)}{(R-(h+y))^2}}$	

Continuation Sheet

Module/Unit:	Advanced Motorcycle Dynamics	Page:	6 of 6
--------------	------------------------------	-------	--------

Question No	Question	Marks
4th cont	$\frac{dy}{dx} = \sqrt{\frac{R^2 - (R - (h-y))^2}{(R - (h-y))^2}}$ $\frac{dy}{dx} = \sqrt{\frac{R^2}{(R - (h-y))^2} - 1}$	(10)
4c	 D gives the peak value C controls the asymptotic value $\rightarrow \infty$ B determines the slope E controls the curve at the lap	(6)
		20